Knot homology invariants and genus 2 mutation

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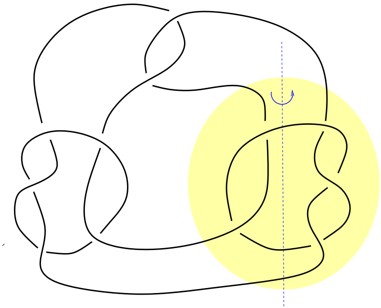


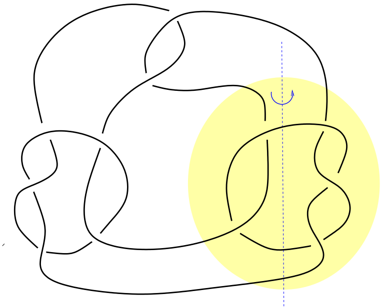
The Kinoshita-Terasaka knot $KT_{2,1}$ as it appears in [4]

Heegaard Floer and Khovanov knot homologies for $K \subset S^3$

$$K \rightsquigarrow \widehat{\text{CFK}}(K) \rightsquigarrow \widehat{\text{HFK}}$$

$$K \rightsquigarrow \text{CKh}(K) \rightsquigarrow \text{Kh}$$

[Ozsváth and Szabó, Rasmussen, Khovanov]



The Kinoshita-Terasak knot $KT_{2,1}$ as it appears in [4]

Theorem (Ozsváth and Szabó [4])

$$\widehat{HFK}(KT_{r,n}) \ncong \widehat{HFK}(C_{r,n})$$

$\widehat{\text{HFK}}(KT)$

	-2	-1	0	1	2
3					$\mathbb{F}$
2				$\mathbb{F}^4$	$\mathbb{F}$
1			$\mathbb{F}^6$	$\mathbb{F}^4$	
0		$\mathbb{F}^4$	$\mathbb{F}^7$		
-1	$\mathbb{F}$	$\mathbb{F}^4$			
-2	$\mathbb{F}$				

$\dim = 33$

$\widehat{\text{HFK}}(C)$

	-3	-2	-1	0	1	2	3
4							$\mathbb{F}$
3						$\mathbb{F}^3$	$\mathbb{F}$
2					$\mathbb{F}^3$	$\mathbb{F}^3$	
1				$\mathbb{F}^2$	$\mathbb{F}^3$		
0			$\mathbb{F}^3$	$\mathbb{F}^3$			
-1		$\mathbb{F}^3$	$\mathbb{F}^3$				
-2	$\mathbb{F}$	$\mathbb{F}^3$					
-3	$\mathbb{F}$						

$\dim = 33$

Observation

$$\bigoplus \dim \widehat{\text{HFK}}(KT) = \bigoplus \dim \widehat{\text{HFK}}(C)$$

$\widehat{\text{HFK}}(KT)$

	-2	-1	0	1	2
3					$\mathbb{F}$
2				$\mathbb{F}^4$	$\mathbb{F}$
1			$\mathbb{F}^6$	$\mathbb{F}^4$	
0		$\mathbb{F}^4$	$\mathbb{F}^7$		
-1	$\mathbb{F}$	$\mathbb{F}^4$			
-2	$\mathbb{F}$				
dim $= 33$					

$\widehat{\text{HFK}}(C)$

	-3	-2	-1	0	1	2	3
4							$\mathbb{F}$
3						$\mathbb{F}^3$	$\mathbb{F}$
2					$\mathbb{F}^3$	$\mathbb{F}^3$	
1				$\mathbb{F}^2$	$\mathbb{F}^3$		
0			$\mathbb{F}^3$	$\mathbb{F}^3$			
-1		$\mathbb{F}^3$	$\mathbb{F}^3$				
-2	$\mathbb{F}$	$\mathbb{F}^3$					
-3	$\mathbb{F}$						
dim $= 33$							

Observation

$$\bigoplus \dim \widehat{\text{HFK}}(KT) = \bigoplus \dim \widehat{\text{HFK}}(C)$$

Questions of rank invariance

This observation belongs to a large body of questions about the rank invariance of homology under different mutations, for both knots and manifolds.

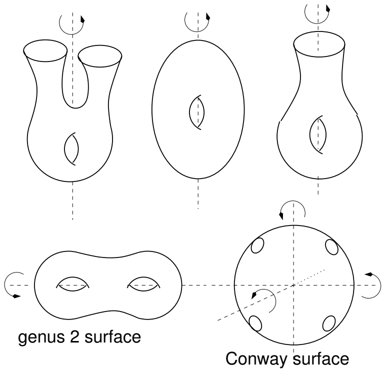
Theorem (M. and Starkston)

There exist infinitely many knots admitting a nontrivial genus 2 mutant with the same total dimension in both knot Floer homology and Khovanov homology.

Genus 2 mutation

M – compact, orientable 3 manifold.

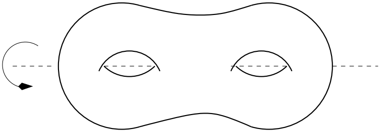
(F, τ) – surface equipped with the hyperelliptic involution.



Genus 2 mutation of a manifold

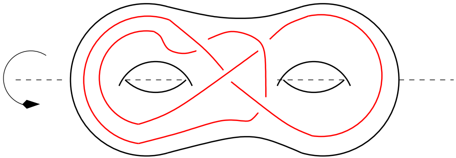
- Cut M along F.
- Involute F by τ.
- Glue in $\tau(F)$.

Resulting manifold is M^τ, a genus 2 mutant of M.



Genus 2 mutation a knot

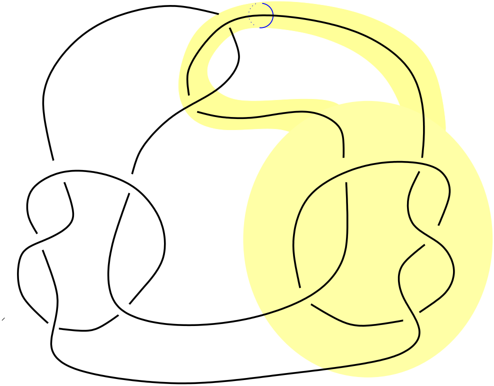
$K, F \subset S^3$. K disjoint from F.
S^3 is simply connected $\Rightarrow F$ is compressible $\Rightarrow S^3 \cong (S^3)^\tau$.



K^τ is the genus 2 mutant of K.

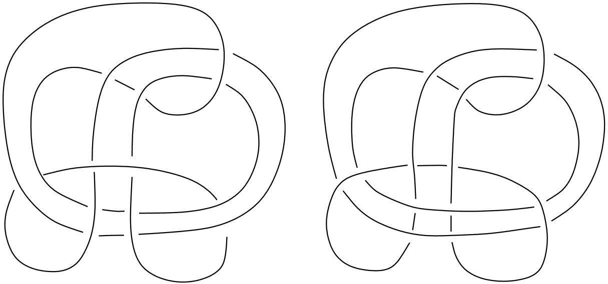
*For us, assume F bounds a handlebody.

Kinoshita-Terasaka/Conway mutants again



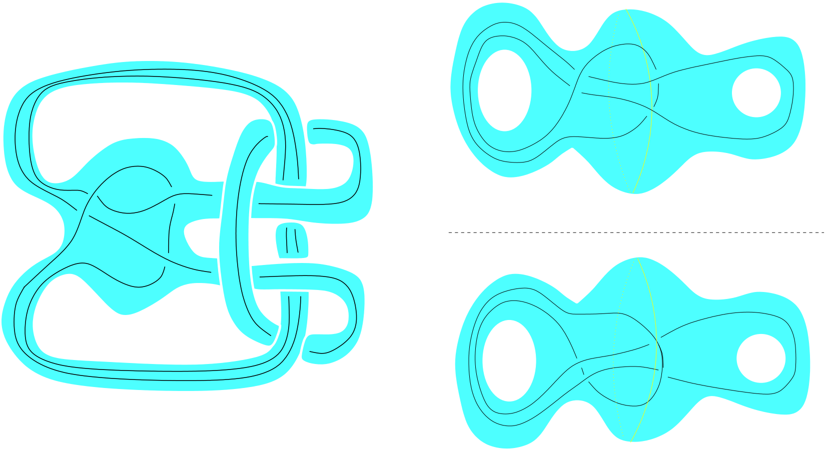
Conway mutants can always be obtained from genus 2 mutation (sometimes requires two mutations).

Genus 2 mutant pair

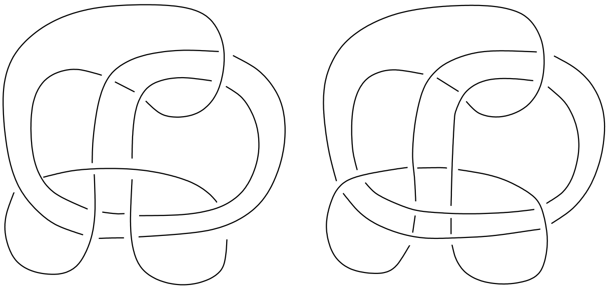


These genus 2 mutant knots are not Conway mutants.

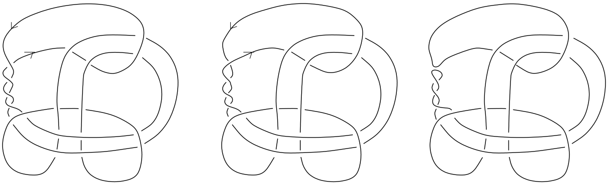
Mutation surface



Genus 2 mutant pair

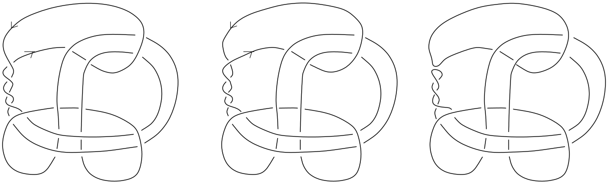


Infinite family parameterized by half-twists



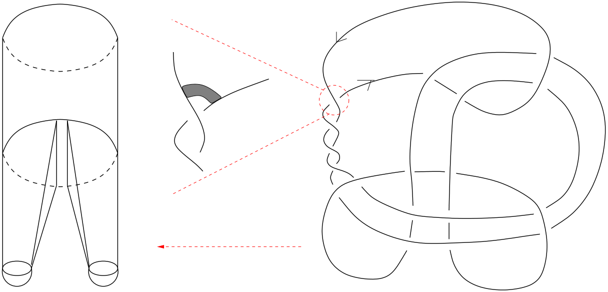
These knots have special properties....

Special property: Skein triples



$$(K_n, \quad K_{n-2}, \quad unlink)$$

Special property: K_n is slice



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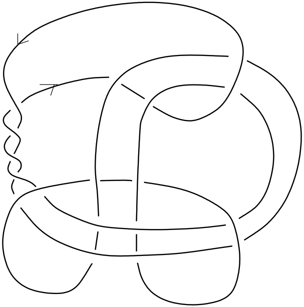

$$|\tau| \leq g_*(K_n) = 0$$

That's Ozsváth and Szabó's τ, not the mutation τ!

What our theorem shows:

$$\widehat{\text{HFK}}(K_n) \overset{}{\underset{\tau \text{ preserves } \oplus \dim}{\rightsquigarrow}} \widehat{\text{HFK}}(K_n^\tau)$$

$$\cong \qquad\qquad\qquad\qquad \cong$$

$$\widehat{\text{HFK}}(K_0) \overset{}{\underset{\tau \text{ preserves } \oplus \dim}{\rightsquigarrow}} \widehat{\text{HFK}}(K_0^\tau)$$

$$\cong \qquad\qquad\qquad\qquad \cong$$

$K_0 = 14^n_{22185}$					
	-2	-1	0	1	2
3					$\mathbb{F}$
2				$\mathbb{F}^2$	$\mathbb{F}$
1			$\mathbb{F}^2$	$\mathbb{F}^2$	
0		$\mathbb{F}^2$	$\mathbb{F}^3$		
-1	$\mathbb{F}$	$\mathbb{F}^2$			
-2	$\mathbb{F}$				
$\dim = 17$					

$K_0^\tau = 14^n_{22589}$			
	-1	0	1
1			$\mathbb{F}^2$
0		$\mathbb{F}^5$	$\mathbb{F}^2$
-1	$\mathbb{F}^2$	$\mathbb{F}^4$	
-2	$\mathbb{F}^2$		
$\dim = 17$			

We need 3 tools from Ozsváth and Szabó. [4],[5]

$$\widehat{HFK}(K_n) \underset{\tau \text{ preserves } \oplus dim}{\rightsquigarrow} \widehat{HFK}(K_n^{\tau})$$

$\cong$ $\cong$

$$\widehat{HFK}(K_0) \underset{\tau \text{ preserves } \oplus dim}{\rightsquigarrow} \widehat{HFK}(K_0^{\tau})$$

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-1	$\mathbb{F}$	$\mathbb{F}^2$			
-2	$\mathbb{F}$				
rank = 17					

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-1	$\mathbb{F}^2$	$\mathbb{F}^4$	
-2	$\mathbb{F}^2$		
rank = 17			

1. $\tau(K_n) = 0$
2. Spectral sequence $\widehat{HFK}(K) \rightsquigarrow \widehat{HF}(S^3)$.
3. Skein exact sequence of $\widehat{HFK}$.

Compare with Hedden, Watson, others [3][6].

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$K_0^\tau = 14^n_{22589}$			
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We need 3 tools from Ozsváth and Szabó. [4],[5]

$$\begin{array}{ccc} \widehat{\mathrm{HFK}}(K_n) & \overset{}{\underset{\tau \text{ preserves } \oplus \dim}{\rightsquigarrow}} & \widehat{\mathrm{HFK}}(K_n^\tau) \\ \cong & & \cong \\ \widehat{\mathrm{HFK}}(K_0) & \underset{\tau \text{ preserves } \oplus \dim}{\rightsquigarrow} & \widehat{\mathrm{HFK}}(K_0^\tau) \\ \cong & & \cong \end{array}$$

$K_0 = 14^n_{22185}$					
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Getting technical:

$$0 \longrightarrow \text{HFK}^-_1(K_n) \xrightarrow{f^-} \text{HFK}^-_1(K_{n-2}) \xrightarrow{g^-} \mathbb{F}_2[U] \xrightarrow{h^-} \text{HFK}^-_0(K_n) \xrightarrow{i^-}$$

$$z_{\{0,0\}} \in \mathbb{F}_2[U], \quad x_{n\{0,0\}} \in \text{HFK}^-_0(K_n), \quad z_{\{0,0\}} \longmapsto x_{n\{0,0\}}$$

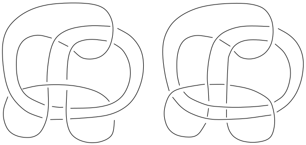
$$\text{HFK}^-_0(K_{n-2}) \xrightarrow{j^-} \mathbb{F}_2[U] \xrightarrow{k^-} \text{HFK}^-_{-1}(K_n) \xrightarrow{\ell^-} \text{HFK}^-_{-1}(K_{n-2}) \longrightarrow 0$$

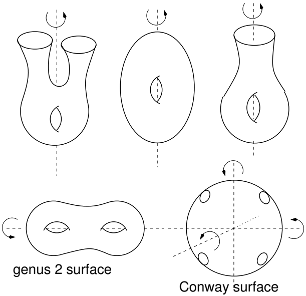
$$x_{n-2\{0,0\}} \in \text{HFK}^-_0(K_{n-2}), \quad z'_{\{-1,0\}} \in \mathbb{F}_2[U], \quad x_{n-2\{0,0\}} \longmapsto z'_{\{-1,0\}}$$

Open questions abound

Manifolds: total rank?

Knots: total rank?





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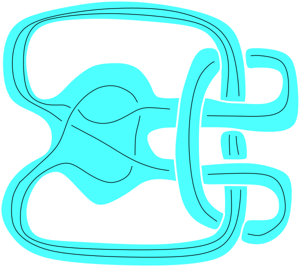
Thank you!



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http://arxiv.org/abs/1204.2524